

$$\int \frac{3x^2}{\sqrt{x^3+1}} dx =$$

## Opener

(A)  $2\sqrt{x^3+1} + C$

(B)  $\frac{3}{2}\sqrt{x^3+1} + C$

(C)  $\sqrt{x^3+1} + C$

(D)  $\ln\sqrt{x^3+1} + C$

(E)  $\ln(x^3+1) + C$

$$\int_2^3 \frac{x}{x^2+1} dx =$$

- (A)  $\frac{1}{2} \ln \frac{3}{2}$       (B)  $\frac{1}{2} \ln 2$       (C)  $\ln 2$       (D)  $2 \ln 2$       (E)  $\frac{1}{2} \ln 5$

If the substitution  $u = \frac{x}{2}$  is made, the integral  $\int_2^4 \frac{1 - \left(\frac{x}{2}\right)^2}{x} dx =$

- (A)  $\int_1^2 \frac{1-u^2}{u} du$       (B)  $\int_2^4 \frac{1-u^2}{u} du$       (C)  $\int_1^2 \frac{1-u^2}{2u} du$   
(D)  $\int_1^2 \frac{1-u^2}{4u} du$       (E)  $\int_2^4 \frac{1-u^2}{2u} du$

## 6-3 day 1 Integration by Parts

### Learning Objectives:

I evaluate an integral using integration by parts.

## Integration by Parts

$$\int u \, dv = uv' - \int v \, du$$

Diagram illustrating the integration by parts formula with annotations:

- A blue circle highlights the  $u$  in the first integral.
- A green circle highlights the  $dv$  in the first integral.
- A blue circle highlights the  $u$  in the second integral.
- A green circle highlights the  $dv$  in the second integral.
- A blue arrow labeled "der" points from the  $dv$  in the first integral to the  $u$  in the second integral.
- A green arrow labeled "A.D." points from the  $u$  in the first integral to the  $dv$  in the second integral.

Ex1. Evaluate

$$\int u dv = u \cdot v - \int v du$$

1.)  $\int x \sin x dx$

$$u = x \longleftrightarrow dv = \sin x dx$$

$$du = 1 dx \longrightarrow v = -\cos x$$

$$\int x \sin x dx = x \cdot (-\cos x) - \int (-\cos x) \cdot 1 dx$$

$$\int x \sin x dx = -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

$$\begin{aligned} 2.) \int x e^x dx & \quad \int u dv = uv - \int v du \\ & \quad u = x \quad dv = e^x dx \\ & \quad \int x e^x dx = x e^x - \int e^x dx = 1 dx \quad v = e^x \\ & \quad \underline{\underline{= x e^x - e^x + C}} \end{aligned}$$

$$3.) \int x^4 \ln x dx$$

$$u = \ln x \longleftrightarrow dv = x^4$$
$$du = \frac{1}{x} \longleftrightarrow v = \frac{1}{5} x^5$$

$$\int \ln x (x^4) dx = (\ln x) \frac{1}{5} x^5 - \int \frac{1}{5} x^{\frac{4}{x}} \frac{1}{x} dx$$
$$= (\ln x) \frac{1}{5} x^5 - \frac{1}{25} x^5 + C$$

$$4.) \int \frac{\ln x}{x^2} dx = \int \frac{1}{x^2} \cdot \ln x dx$$
$$= \int x^{-2} \cdot \ln x dx$$

$$u = \ln x \quad dv = x^{-2}$$
$$du = \frac{1}{x} \quad v = -x^{-1}$$

$$\rightarrow = -x^{-1} \cdot \ln x - \int -x^{-1} \cdot \frac{1}{x} dx$$
$$= -\frac{\ln x}{x} + \int \frac{1}{x^2} dx \quad x^{-2}$$
$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

Ex2. Evaluate  $\int_1^4 \sqrt{t} \ln t \, dt$        $\int_1^4 t^{1/2} \ln t \, dt$

$$u = \ln t$$

$$du = \frac{1}{t}$$

$$dv = t^{1/2}$$

$$v = \frac{2}{3} t^{3/2}$$

$$\begin{aligned} \int_1^4 t^{1/2} \ln t \, dt &= \ln t \cdot \frac{2}{3} t^{3/2} - \int_1^4 \frac{2}{3} t^{3/2} \cdot \frac{1}{t} \, dt \\ &= \frac{2}{3} t^{3/2} \ln t - \frac{2}{3} \int_1^4 t^{1/2} \, dt \\ &= \frac{2}{3} t^{3/2} \ln t - \frac{2}{3} \cdot \frac{2}{3} t^{3/2} \Big|_1^4 \\ &= \left[ \frac{2}{3} t^{3/2} \ln t - \frac{4}{9} t^{3/2} \right]_1^4 \\ &= \left[ \frac{2}{3} \cdot 4^{3/2} \ln 4 - \frac{4}{9} \cdot 4^{3/2} \right] - \left[ \frac{2}{3} \cdot 1^{3/2} \ln 1 - \frac{4}{9} \cdot 1^{3/2} \right] \\ &= \left[ \frac{2}{3} \cdot 8 \ln 4 - \frac{4}{9} \cdot 8 \right] - \left[ 0 - \frac{4}{9} \right] \\ &= \frac{16}{3} \ln 4 - \frac{32}{9} + \frac{4}{9} \\ &= \boxed{\frac{16}{3} \ln 4 - \frac{28}{9}} \end{aligned}$$

## Homework

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